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Boundry Layer Flow and Heat Transfer along an Infinite Porous Hot Horizontal Continuous Moving Plate By Means Of the Natural Transformation Method

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Abstract
An analysis is made for the two-dimensional laminar boundary layer flow of a viscous, incompressible fluid,along an infinite porous hot horizontal continuous moving plate. The governing system of partial differential equations was transformed into ordinary differential equations before being solve by Natural transformation. The effects of various physical parameters, such as Eckert number Ec, Prandtlnumber $\Pr$, plate velocity α and heat source/sink parameter Sare presented and discussed. It is found that the plate velocity significantly effects the flow and heat transfer.

Keywords:boundary layer flow, moving plate, laminar flow, heat transfer, natural transform.

Nomenclature

C_f	Skin friction coefficient
Ec	Eckert number
Nu	Nusselt number
Pr	Prandtl number
Q	Volumetric rate of heat generation
S	Heat source/sink parameter
T	Temperature of the fluid
T_w	Temperature at the wall
T_∞	Free stream temperature
u, v	Velocity component of the fluid along the x and y directions, respectively
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

ρ	Density of the fluid
μ	Viscosity of the fluid
κ	Thermal conductivity
υ	Kinematic viscosity
θ	Dimensionless temperature

Superscript

$'$	Derivative with respect to y

Subscripts

w	Properties at the plate
∞	Free stream condition

I. Introduction

The study of boundary layer flow over a solid surface has generated much interest in recent years in view of its significant applications in industrial manufacturing such as the aerodynamic extrusion of plastic sheets, the boundary layer along material handling conveyers, the cooling of an infinite metallic plate in a cool bath, and the boundary layer along a liquid film in condensation processes. The classical problem was introduced by Blasius [4] where he considered the boundary layer flow on a fixed flat plate. Different from Blasius [4], in his pioneering work, Sakiadis [12] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [6] extended the work of Sakiadis [12] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface.

The Natural transform, initially was defined by Khan and Khan [7] as N - transform, who studied their properties and applications. Later ,Belgacem et al. [3] and Silambarasan et al. [14] defined its inverse and studied some additional fundamental properties of this integral transform and named it the Natural transform. Applications of Natural transform in the solution of differential and integral equations and for the distribution and Boehmians spaces can be found in [1, 5, 7,8, 9, 11].

The goal of the present investigation is to provide a robust algorithm for the solution of the two-dimensional laminar boundary layer flow of a viscous incompressible fluid, along an infinite porous hot horizontal continuous moving plate using Natural transformation and discuss effect of various parameters on fluid flow and heat transfer.

II. Formulation of the problem:

Let us consider two dimensional laminar steady boundary layer flow and heat transfer of a viscous incompressible fluid along an infinite hot continuous moving flat plate in the presence of constant suction at the surface, constant free stream U_∞ and heat generation (or absorption). The plate is moving in flow direction with constant velocity U_W and maintain at constant temperature T_W, where x-axis is along the flow and y-axis perpendicular to it.. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy(Pai [10], Schlichting [13], Bansal [2]) are:

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow \quad v^* = -v_0 \;\; (\text{constant}), \;\; v_0 > 0 \quad \dots(1)$$

$$\rho\left(-v_0 \frac{\partial u^*}{\partial y^*}\right) = \mu \frac{\partial^2 u^*}{\partial y^{*2}} \quad \dots(2)$$

$$\rho C_p\left(-v_0 \frac{\partial T^*}{\partial y^*}\right) = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left(\frac{\partial u^*}{\partial y^*}\right)^2 + Q\left(T^* - T_\infty\right) \quad \dots(3)$$

Along which the boundary conditions are:

$$\begin{aligned} y^* = 0 \;\; &: u^* = U_w, v^* = -v_0, T^* = T_w \\ y^* \to \infty &: u^* \to U_\infty, T^* \to T_\infty \end{aligned} \quad \dots(4)$$

III. Analysis

The momentum and energy equations can be transformed into the corresponding ordinary differential equation by using the following non-dimensional parameters:

$$\left.\begin{aligned} & y = y^* \frac{v_0}{\upsilon}, u = \frac{u^*}{U_\infty}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \alpha = \frac{U_w}{U_\infty} \\ & \Pr = \frac{\mu C_p}{\kappa}, S = \frac{Qv^2}{\kappa v_0^2}, Ec = \frac{U_\infty^2}{C_p\left(T_w - T_\infty\right)} \end{aligned}\right\} \dots(5)$$

The transformed ordinary differential equations are:

$$u'' + u' = 0 \quad \dots(6)$$

$$\theta'' + \Pr\theta' + S\theta = -Ec\Pr(u')^2 \quad \dots(7)$$

The transformed boundary conditions are:

$$\left.\begin{aligned} y = 0:&\;\; u = \alpha, \theta = 1 \\ y \to \infty:&\;\; u \to 1, \theta \to 0 \end{aligned}\right\} \quad \dots(8)$$

where prime denotes differentiation with respect to y, $Pr = \frac{\mu c_p}{\kappa}$ is the Prandtl number, α is the velocity of the plate, $S = \frac{Qv^2}{\kappa v_0^2}$ is the heat source (S<0) or sink (S>0) parameter and $Ec = \frac{U_\infty^2}{C_p(T - T_\infty)}$ is the Eckert number.

For the solution of equations (6) and (7) subject to boundary condition (8), using Natural transform, we have:

$$\bar{u}(s,c) = \frac{\alpha}{s+c} + \frac{\alpha}{c}\left\{u'(0) + \alpha\right\}\left\{\frac{1}{s} - \frac{1}{s+c}\right\} \quad \dots(9)$$

where $\bar{u}(s,c) = N^+\left[u(y)\right]$

$$\bar{\theta}(s,c) = \frac{K}{s+2c} + (1-K)\left[\frac{s + \frac{c\Pr}{2}}{\left(s + \frac{c\Pr}{2}\right)^2 - \left(\frac{c^2\Pr^2}{4} - c^2 S\right)}\right] + \frac{c\left\{K\left(2-\Pr\right) + \theta'(0) + \Pr + (K-1)\frac{\Pr}{2}\right\}}{\left(s + \frac{c\Pr}{2}\right)^2 - \left(\frac{c^2\Pr^2}{4} - c^2 S\right)} \quad \dots(10)$$

where $\bar{\theta}(s,c) = N^+\left[\theta(y)\right]$ and

$$K = -\frac{Ec\Pr(\alpha-1)^2}{(4 - 2\Pr + S)}$$

Now inverting the equations (9) and (10) by inverse Natural transform, with the corresponding initial and boundary conditions:

$$u(y) = 1 + (\alpha - 1)e^{-y} \quad \dots(11)$$

$$\theta(y) = Ke^{-2y} + (1-K)e^{\frac{-\Pr - \sqrt{\Pr^2 - 4S}}{2}y} \quad \dots(12)$$

where $K = -\dfrac{Ec\Pr(\alpha-1)^2}{(4 - 2\Pr + S)}$ $\quad \dots(13)$

IV. Skin friction and Nusselt number:

Having known the velocity and temperature fields we can now obtain the expression for the dimensionless Skin friction C_f and Nusselt number Nu, which are given by

$$\left(C_f\right)_{y=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = 1 - \alpha \quad \dots(14)$$

and

$$\mathrm{Nu} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = 2K + (1-K)\left\{\frac{-Pr - \sqrt{Pr^2 - 4S}}{2}\right\} \quad \dots(15)$$

The numerical results of C_f and Nu are shown in table 1 and 2 respectively.

V. Results and Discussion

The present solution of the problem describes the general feature of boundary layer flow and heat transfer along an infinite porous hot horizontal continuous plate. The governing equations (6)-(7) together with the boundary conditions (8) are solved using Natural transform method. Table 1 represents the Skin friction parameter C_f against the plate velocityα. It is noted that the effect of increasing plate velocity is to decrease the Skin friction at the surface (keeping free stream velocity same).

Table 1. Numerical values of Skin friction C_f for different values of α

α	C_f
0.0	1.0
0.5	0.5
1.0	0.0
1.5	-0.5
2.0	-1.0
2.5	-1.5

Table 2 represents the Nusselt number Nu for various values of Eckert number Ec, Prandtl number Pr, plate velocity α and heat source/sink parameter S. It is observed that the rate of heat transfer parameter i.e. Nusselt number is an increasing function ofEc, Pr, α and S. The physical meaning being that, when plate velocity exceeds the free stream velocity.

Table 2. Numerical values of Nusselt number Nu for different values of Ec, Pr, α and S

Ec	Pr	α	S	Nu
0.01	1.0	0.0	0.0	0.9950
0.01	1.0	0.0	0.1	0.8820
0.01	2.0	1.0	0.1	1.9487
0.01	3.0	1.0	0.1	2.9663
0.01	4.0	2.0	0.0	3.9800
0.02	2.0	1.5	0.4	1.7690
0.02	3.0	2.5	0.4	2.7876
0.02	4.0	2.5	0.3	3.8300
0.02	5.0	1.5	0.5	4.8847
0.03	1.5	2.0	0.2	1.3278
0.01	2.5	2.0	0.3	2.3469
0.02	1.0	1.5	0.2	0.7207
0.03	3.5	0.0	0.3	3.3572

Figure 1 shows the effect of plate velocity α on velocity profile. We infer from this figure that velocity increases considerably as plate velocity α increases. Moreover, when $0 < \alpha < 1$ (that is when plate velocity is less than free stream velocity), the profile for u is concave down, and when $\alpha > 1$ (that is when free stream velocity less than plat velocity), the profile for u is concave up.

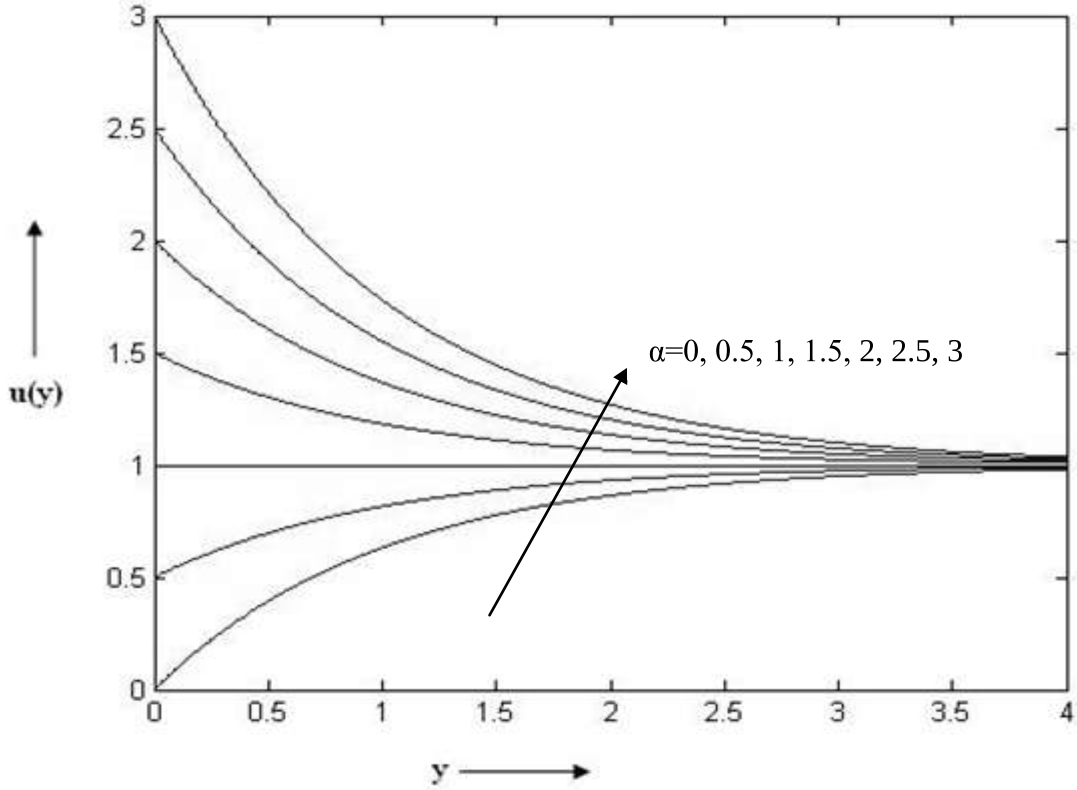
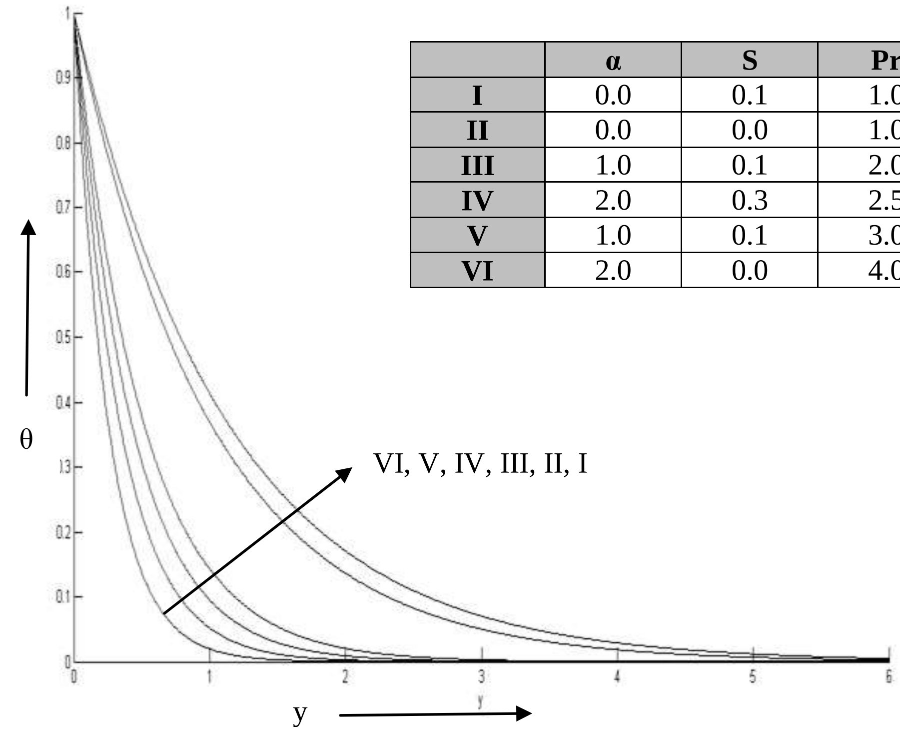


Figure 1: Velocity profile for different values of platevelocity α.

Figure 2 is illustrate the temperature profile for various values of Prandtlnumber Pr, plate velocity α and heat source/sink parameter S. We infer from this figure that the values ofθ increase with the increasingS, but reversed phenomenon is observed for increasing values of αand Pr.



	α	S	Pr
I	0.0	0.1	1.0
II	0.0	0.0	1.0
III	1.0	0.1	2.0
IV	2.0	0.3	2.5
V	1.0	0.1	3.0
VI	2.0	0.0	4.0

Figure 2: Temperature profile for different values of α, S and Pr, when Ec=0.01.

VI. Conclusion

In the present study, we have theoretically studied the boundary layer flow and heat transfer along an infinite porous hot horizontal continuous plate problem. The governing equations together with the boundary condition are solved using Natural transform method. The velocity and temperature profiles are discussed numerically and presented through graphs. The numerical values of Skin friction and Nusselt number are derived, discussed numerically and presented through tables. From the study, following conclusions can be drawn:

(i) The mean velocity profile is concave down for $0 < \alpha < 1$ and is concave up for $\alpha > 1$.
(ii) The mean temperature profile increase with the increasing S, but reversed phenomenon is observed for increasing values ofα andPr.
(iii) The effect of increasing plate velocity is to decrease the Skin friction.
(iv) Nusselt number is an increasing function of Ec, Pr, α and S.

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